Titles and Abstracts

Mortar coupling of hp-FEM and hp-BEM for the Helmholtz equation

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Many wave propagation problems are posed in unbounded domains. An established technique to discretize problems in unbounded domains is to introduce an artificial interface that decomposes the unbounded domain into a bounded part and the exterior unbounded part. Assuming a homogeneous medium in the unbounded part, a discretization is then obtained by coupling a FEM discretization in the bounded part with a boundary integral method (BEM) for the unbounded exterior. Here, we employ the high order discretizations hp-FEM and hp-BEM of piecewise polynomials of degree p on meshes of size h.

For the time-harmonic Helmholtz equation at large wavenumber k, we present a FEM-BEM coupling procedure and a k-explicit error analysis of this method. That is, for a heterogeneous medium in the bounded part and a homogeneous medium in the unbounded part and under analyticity assumptions on the interfaces we show that our FEM-BEM coupling is quasioptimal under the conditions that a) kh/p is sufficiently small and b) $p = O(\log k)$.

The conditions on h and p are those established earlier for hp-FEM for various Helmholtz problems, [MS11, MSP12, BCFM25] or hp-BEM, [LM]. As in these works, the key is a k-explicit regularity theory that decomposes the solutions into two components: the first component is a piecewise analytic, but highly oscillatory function and the second one has finite regularity but features wavenumber-independent bounds. En route various integral operators are decomposed into parts that map into spaces of analytic functions and Sobolev spaces of finite regularity with norm bounds explicit in k.

Our mortar FEM-BEM coupling relies on so-called combined field discretizations well-established for exterior scattering problems and reduces to the classical "symmetric coupling" in the low-frequency limit $k \to 0$. Our analysis can be extended to $hp - DG$ discretizations in the bounded part.

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Preasymptotic analyses of FEM and CIP-FEM for the Helmholtz equation with large wave number

Haijun Wu(武海军)
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Several aspects of the FEM and the CIP-FEM for the Helmholtz equation with large wave number are discussed when the mesh size in the preasymptotic regime, including a priori error estimates, a posteriori error estimates, convergence and quasi-optimality of the adaptive algorithm, and so on.

Stable approximation of Helmholtz solution with evanescent plane waves

Andrea Moiola
University of Pavia

Helmholtz solutions can be accurately approximated by linear combinations of propagative plane waves (PPWs). However, this approximation is unstable: to approximate some smooth Helmholtz solutions, PPWs require huge coefficients. In computer arithmetic, this leads to numerical cancellation and prevents any accuracy.

A remedy to such instability is the use of evanescent plane waves (EPWs): plane waves with complex propagation vectors. Any Helmholtz solution on a ball can be written as a continuous superposition of EPWs with bounded coefficient density. We propose a discretization strategy of this representation, based on modern sampling techniques, to approximate any solution with a finite combination of EPWs with bounded coefficients.

The theory is supported by numerical experiments on the ball and on convex shapes, including the application to Trefftz discontinuous Galerkin (TDG) schemes. We also derive an integral representation of any Helmholtz solution in smooth convex 2D domains as a superposition of EPWs and PPWs.

This is a joint work with Nicola Galante (INRIA Paris), Daan Huybrechs (KU Leuven), and Emile Parolin (INRIA Paris).

Does PML exponentially absorb outgoing waves scattering from a periodic surface?

Wangtao Lu(鲁汪涛)
Zhejiang University

The PML method is well-known for its exponential convergence rate and easy implementation for scattering problems with unbounded domains. For rough-surface scattering problems, it was proved that the PML method converges at most algebraically in the physical domain. However, the question whether exponential convergence still holds for compact subsets remains unclear. In a previous work, one of our authors proved the exponential convergence for 2π-periodic surfaces via the Floquet-Bloch transform when $2k \in \mathbb{R}^+ \backslash \mathbb{Z}$ where k is the wavenumber; when $2k \in \mathbb{R}^+ \cap \mathbb{Z}$, a nearly fourth-order convergence rate was shown later on. The extension of this method to locally perturbed cases is not straightforward, since the domain is no longer periodic thus the Floquet-Bloch transform doesn't work, especially when the domain topology is changed. Moreover, the exact decay rate when $2k \in \mathbb{R}^+ \cap \mathbb{Z}$ remains unclear. The purpose of this paper is to address these two significant issues. For the first topic, the main idea is to reduce the problem by the DtN map on an artificial curve, then the convergence rate of the PML is obtained from the investigation of the DtN map. It shows exactly the same convergence rate as in the unperturbed case. Second, to illustrate the convergence rate when $2k \in \mathbb{R}^+ \cap \mathbb{Z}$, we design a specific periodic structure for which the PML converges at the fourth-order, showing that the algebraic convergence rate is sharp. We adopt a previously developed high-accuracy PML-BIE solver to exhibit this unexpected phenomenon.

Convergence of overlapping domain decomposition methods with PML transmission conditions applied to nontrapping Helmholtz problems

David Lafontaine
CNRS and Institut de Mathématiques de Toulouse

We will be interested in overlapping Schwarz methods for the Helmholtz equation with real (large) wavenumber and smooth variable wave speed, where the radiation condition is approximated by a Cartesian perfectly-matched layer (PML), the domain-decomposition subdomains are overlapping hyperrectangles with Cartesian PMLs at their boundaries, and the overlaps of the subdomains and the widths of the PMLs are all independent of the wavenumber.
For both parallel (i.e., additive) and sequential (i.e., multiplicative) methods, we have recently shown that after a specified number of iterations – depending on the ray dynamics – the error is smooth and smaller than any negative power of the wavenumber. For the parallel method, the specified number of iterations is less than the maximum number of subdomains, counted with their multiplicity, that a geometric-optic ray can intersect.
The main goal of our talk will be to give the main ideas behind the proof of this result, based on propagation of singularities for high frequency Helmholtz solutions. It will be complementary to Shihua Gong's talk, which will delve into numerics and implementation, while we focus on the theoretical aspects.
This is a joint work with Jeffrey Galkowski, Shihua Gong, Ivan Graham, and Euan Spence.

Wave-number-explicit analysis for time-harmonic wave equations with Dirichlet-to-Neumann truncation

Yonglin Li(李勇霖)
Wuhan University

This topic is focused on the propagation of waves with large wave numbers and Dirichlet-to-Neumann (DtN) truncations. The model problem is approximated by truncating the exact DtN operator into a finite sum of expansions. We prove the well-posedness and wave-number-explicit stability of the solution to truncated problem by assuming that the truncation number N satisfies $N \geq \lambda kR$ for some $\lambda>1$, where k represents the wave number and R is the radius of the physical domain. Additionally, we show that the truncated solution is exponentially close to the true scattering solution in terms of N. The finite element methods (FEM) for the truncated problems, along with the (pre-)asymptotic error estimates, are presented. Some numerical experiments are provided to validate the theoretical findings.

An efficient multiscale generalised FEM for high-frequency wave propagation in heterogeneous media

Robert Scheichl
Heidelberg University

In this talk, I will present a novel generalized finite element method with optimal local approximation spaces for high-frequency wave propagation problems in heterogeneous media. The local spaces are built from selected eigenvectors of carefully designed local eigenvalue problems defined on generalized harmonic spaces. The robustness and efficiency of the approach is studied systematically in the context of the Helmholtz equation. At both the continuous and the discrete level, (i) wavenumber explicit and nearly exponential decay rates for local and global approximation errors are obtained without any assumption on the size of subdomains and (ii) a quasi-optimal convergence of the method is established by assuming that the size of subdomains is O(1/k), where k is the wavenumber. A novel resonance effect between the wavenumber and the dimension of local spaces on the decay of error with respect to the oversampling size is implied by the analysis. At the continuous level the method extends the plane wave partition of unity method [I. Babuška and J. M. Melenk, Internat. J. Numer. Methods Engrg., 40 (1997), pp. 727–758] to the heterogeneous-coefficients case, and at the discrete level, it delivers an efficient localised model reduction method, as well as an effective coarse space for a two-level restricted additive Schwarz (RAS) preconditioner (see Chupeng Ma's talk). Numerical results are provided to confirm the theoretical analysis and to demonstrate the efficiency and robustness of the proposed method. This is joint work with Christian Alber (Heidelberg) and Chupeng Ma (Great Bay University).

Localized orthogonal decompositions for high-frequency Helmholtz problems

Daniel Peterseim
Universität Augsburg

We discuss old and new variants of the Localized Orthogonal Decomposition (LOD) method for time-harmonic scattering problems of Helmholtz type with high wavenumber. On a coarse mesh, the method identifies local finite element source terms that yield rapidly decaying responses under the solution operator. These responses can be constructed to high accuracy from independent local snapshot solutions on patches and are used as problem-adapted basis functions in the method. The associated localization error decays at least exponentially as the oversampling parameter is increased. Optimal convergence of resulting methods is observed under some minimal resolution condition. Numerical experiments demonstrate the offline and online performance of the method also in the case of heterogeneous media and perfectly matched layers.

Numerical homogenization for time-harmonic Maxwell equations in heterogeneous media with large wavenumber

Guanglian Li(李光莲)
The University of Hong Kong

We propose a new numerical homogenization method based upon the edge multiscale method for time-harmonic Maxwell equations in heterogeneous media with large wavenumber. Numerical methods for time-harmonic Maxwell equations in homogeneous media with large wavenumber is very challenging due to the so-called pollution effect: the mesh size should be much smaller than the reciprocal of the wavenumber to obtain a solution with certain accuracy. It is much more challenging for the case with heterogeneous media that occurrs often in the practical applications, such as the simulation of metamatrial, since one has to resolve the heterogeineouty for a reasonable numerical solution. We devise a novel approach that do not to resolve the heterogeneouty in the coefficient and has a mesh size linearly depends on the reciprocal of the wavenumber, which has a first order convergence rate. Extensive numerical tests are provided to verify our theoretical findings.

Finite element approximations for Maxwell variational problems on curved domains: domain and quadrature effects

Carlos Jerez-Hanckes
Universidad Adolfo Ibanez

We investigate key challenges in finite element approximations for Maxwell variational problems on curved domains, focusing on two critical aspects: domain approximation and the influence of numerical quadrature. When exact parametrization of curved domains is unavailable, domain approximation relies on polyhedral meshes that only approximate the geometry. We establish sufficient conditions on mesh quality to ensure optimal error convergence rates between discrete solutions on approximate domains and continuous solutions in the original domain. Additionally, we analyze the impact of numerical quadrature rules on solution accuracy for Maxwell problems, considering both straight and curved elements in polygonal and curved domains. For curved domains, we isolate and quantify the quadrature-induced error, providing sufficient conditions on quadrature precision and mesh refinement to preserve expected convergence rates. Numerical results validate our theoretical findings and highlight the interplay between domain approximation and quadrature precision in achieving reliable and accurate finite element solutions.

Some convergence results for RAS-Imp and RAS-PML for the Helmholtz equation

Shihua Gong(龚世华)
The Chinese University of Hong Kong, Shenzhen/SICIAM

We consider two variants of restricted overlapping Schwarz methods for the Helmholtz equation. The first method, known as RAS-Imp, incorporates impedance boundary condition to formulate the local problems. The second method, RAS-PML, employs local perfectly matched layers (PML). These methods combine the local solutions additively with a partition of unity. We have shown that RAS-Imp has power contractivity for strip domain decompositions. More recently, we have shown that RAS-PML has super-algebraic convergence with respect to wavenumber after a specified number of iterations. In this talk we review these results and then investigate their sharpness using numerical experiments. We also investigate situations not covered by the theory. In particular, the theory needs the overlap of the domains or the PML widths to be independent of k. We present numerical experiments where this distance decreases with k. This is a joint work with Jeffrey Galkowski, Ivan Graham, David Lafontaine and Euan Spence.

Substructured non-overlapping DDM vs. ORAS for large scale Helmholtz problems with multiple sources

Christophe Geuzaine
University of Liège

Solving large scale Helmholtz problems discretized with high-order finite elements is notoriously difficult, especially in 3D where direct factorization of the system matrix is very expensive and memory demanding, and robust convergence of iterative methods is difficult to obtain. Domain decomposition methods (DDM) constitute one of the most promising strategy so far, by combining direct and iterative approaches: using direct solvers on overlapping or non-overlapping subdmains, as a preconditioner for a Krylov subspace method on the original Helmholtz system or as an iterative solver on a substructured problem involving field values or Lagrange multipliers on the interfaces between the subdomains. In this talk we will compare non-overlapping substructured DDM and Optimized Restricted Additive Schwarz (ORAS) preconditioners for solving large scale Helmholtz problems with multiple sources, as is encountered e.g. in frequency-domain Full Waveform Inversion.

On the theory of two-level hybrid Schwarz preconditioners for the high-frequency Helmholtz equation

Euan A. Spence
University of Bath

We consider two-level hybrid Schwarz domain-decomposition preconditioners for GMRES applied to finite-element discretisations of the Helmholtz equation with wavenumber k. We analyse this set up under the assumptions that (i) the coarse problem is quasi-optimal, with quasi-optimality constant independent of k, and (ii) the subdomain widths are a sufficiently small multiple of k^{-1} (so that the subdomain problems are coercive) and the subdomain have generous overlap.
We prove that, under these assumptions, the field of values of the preconditioner is bounded above and below (away from the origin) independently of both k and the number of subdomains N, so that (by the Elman estimate) GMRES converges in a number of iterations independent of both k and N.
We apply this result when the fine space consists of piecewise-polynomials of fixed degree and the coarse space consists of piecewise-polynomials of higher degree, such that the ratio of the dimension of the coarse space to the dimension of the fine space tends to zero as k goes to infinity.
This is joint work with Ivan Graham.

A two-level RAS preconditioner based on multiscale spectral generalized FEM for heterogeneous Helmholtz problems

Chupeng Ma(马楚鹏)
Great Bay University

In this talk, I will present a two-level restricted additive Schwarz (RAS) preconditioner for heterogeneous Helmholtz problems, based on a multiscale spectral generalized finite element method (MS-GFEM) proposed in [C. Ma, C. Alber, and R. Scheichl, SIAM. J. Numer. Anal., 61 (2023), pp. 1546--1584]. The preconditioner uses local solves with impedance boundary conditions, and a global coarse solve based on the MS-GFEM approximation space constructed from local eigenproblems. When used within GMRES, it converges at a rate of Λ under some reasonable conditions, where Λ denotes the error of the underlying MS-GFEM approximation. In particular, Λ is not only independent of the fine-mesh size h and the number of subdomains, but decays with increasing wavenumber k. Extensive numerical experiments will be provided to verify the effectiveness of the preconditioner.

Two-level hybrid Schwarz methods for Helmholtz equation with high wavenumber

Peipei Lu (卢培培)[1] and Bowen Zheng (郑博文)[2]
[1] Soochow University
[2] Tongji University

In this talk, we discuss and analyze two-level hybrid Schwarz preconditioners for solving the Helmholtz equation with high wave number in two and three dimensions. Both preconditioners are defined over a set of overlapping subdomains, with each preconditioner formed by a global coarse solver and one local solver on each subdomain. The global coarse solver is based on the localized orthogonal decomposition (LOD) technique, which was proposed originally for the discretization schemes for elliptic multiscale problems with heterogeneous and highly oscillating coefficients and Helmholtz problems with high wave number to eliminate the pollution effect. The local subproblems are Helmholtz problems in subdomains with homogeneous boundary conditions (the first preconditioner) or impedance boundary conditions (the second preconditioner). Both preconditioners are shown to be optimal under some reasonable conditions, that is, a uniform upper bound of the preconditioned operator norm and a uniform lower bound of the field of values are established in terms of all the key parameters, such as the fine mesh size, the coarse mesh size, the subdomain size and the wave numbers. It is the first time to show that the LOD solver can be a very effective coarse solver when it is used appropriately in the Schwarz method with multiple overlapping subdomains. Numerical experiments are presented to confirm the optimality and efficiency of the two proposed preconditioners. The global coarse solvers involved in the two preconditioners can be quite expensive for very large wave numbers. At the end of the talk, we will explore the possibilities to reduce the complexity of the global coarse solvers.

Resolving divergence: the first multigrid scheme for the highly indefinite Helmholtz equation using classical components

Kees Vuik
Delft University of Technology

In this talk, we present the first stand-alone classical multigrid solver for the highly indefinite 2D Helmholtz equation with constant costs per iteration, addressing a longstanding open problem in numerical analysis [1]. Our work covers both large constant and nonconstant wavenumbers up to $k = 500$ in 2D.
We obtain a full $V-$ and W-cycle without any level-dependent restrictions. Another powerful feature is that it can be combined with the computationally cheap weighted Jacobi smoother. The key novelty lies in the use of higher-order inter-grid transfer operators [2]. When combined with coarsening on the Complex Shifted Laplacian, rather than the original Helmholtz operator, our solver is h-independent and scales linearly with the wavenumber k. If we use GMRES(3) smoothing

we obtain k - independent convergence, and can coarsen on the original Helmholtz operator, as long as the higher-order transfer operators are used.
This work opens doors to study robustness of contemporary solvers, such as machine learning solvers inspired by multigrid components, without adding to the black-box complexity.

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Frequency-adaptive multi-scale deep neural networks

Jizu Huang(黄记祖)
Chinese Academy of Sciences

Multi-scale deep neural networks (MscaleDNNs) with downing-scaling mapping have demonstrated superiority over traditional DNNs in approximating target functions characterized by high frequency features. However, the performance of MscaleDNNs heavily depends on the parameters in the downing-scaling mapping, which limits their broader application. In this work, we establish a fitting error bound to explain why MscaleDNNs are advantageous for approximating high frequency functions. Building on this insight, we construct a hybrid feature embedding to enhance the accuracy and robustness of the downing-scaling mapping.
To reduce the dependency of MscaleDNNs on parameters in the downing-scaling mapping, we propose frequency-adaptive MscaleDNNs, which adaptively adjust these parameters based on a posterior error estimate that captures the frequency information of the fitted functions. Numerical examples, including wave propagation and the propagation of a localized solution of the Schrödinger equation with a smooth potential near the semi-classical limit, are presented.
These examples demonstrate that the frequency-adaptive MscaleDNNs improve accuracy by two to three orders of magnitude compared to standard MscaleDNNs.

Fast algorithms for wave scattering and inverse scattering based on boundary integral equations

Jun Lai(赖俊)
Zhejiang University

Wave scattering and inverse scattering have important applications in geophysical exploration, radar detection, medical imaging, and nondestructive testing, etc.. How to achieve fast solutions to the wave equation is one of the widely concerned issues in computational mathematics. Integral equations provide an effective computational tool for solving wave scattering and inverse scattering, especially for problems in unbounded regions. However, in numerical implementation, the integral equation method needs to overcome difficulties such as numerical discretization of singular integrals and solving dense linear matrices. In this talk, we will address this issue for wave scattering in complex structures, combining integral equations, scattering matrices, and the Fast Multipole Method (FMM), to design efficient algorithms for wave scattering problems. Furthermore, by using time reversal theory, we will develop fast imaging algorithms for inverse scattering in the case of multiple particles.

Fourier analysis of finite difference methods for the Helmholtz equation

Hui Zhang(张辉)
Xi'an Jiaotong-Liverpool University

Inspired by the work of Dwarka and Vuik on the pollution error, we show that Fourier analysis can be used as a quantitative tool for estimating the accuracy of finite difference methods for the Helmholtz equation. In particular, for the classical 3-point central scheme for the 1D Dirichlet problem, we show rigorously the exact order of convergence in H^1-norm is k^3h^2. Previously, Babuska, Sauter et al have established similar results for the linear FEM. We show sharpness of the order with two-sided bounds, and find also the order of relative errors for nonzero source problems. As a visual tool, Fourier analysis make it straightforward to draw conclusions from the curve of error against frequency. For example, we use it to compare the accuracy of some optimized finite difference methods.

High-frequency time-harmonic waves, directional sweeping and multipreconditioning

Niall Bootland
Rutherford Appleton Laboratory

We consider the use of multipreconditioning, which allows for multiple preconditioners to be applied in parallel, on high-frequency Helmholtz problems. We take inspiration from domain decomposition strategies known as sweeping methods, which have gained notable interest for their ability to yield nearly-linear asymptotic complexity and which can also be favourable for high-frequency problems. While successful approaches exist, such as those based on higher-order interface conditions, perfectly matched layers (PMLs), or complex tracking of wave fronts, they can often be quite involved or tedious to implement. We investigate here the use of simple sweeping techniques applied in different directions which can then be incorporated in parallel into a multipreconditioned GMRES strategy. Preliminary numerical results on a two-dimensional benchmark problem will demonstrate the potential of this approach.

Adaptive nonoverlapping preconditioners for the Helmholtz equation

Yi Yu(于毅)
Guangxi University

One of the issues with traditional preconditioning of the Helmholtz equations is the potential ill-posedness of the local Dirichlet boundary problem. In this talk, we introduce a new iterative substructuring method, which is similar in concept to the Schur complement system used for elliptic problems. This new structure ensures the well-posedness of the local Dirichlet problems by incorporating the small-magnitude eigenvalues from each subdomain into the coarse problem. Another key challenge of traditional preconditioning of the Helmholtz equations lies in constructing an effective coarse space for non-overlapping methods. Motivated by the success of using generalized eigenvalue problems to precondition elliptic equations with heterogeneous coefficients, we propose two types of DDMs that construct a robust coarse problem. Moreover, our construction is purely algebraic, facilitating straightforward extension to other discretizations and the case of heterogeneous Helmholtz coefficients, while convergence theorems remain valid when the thresholds are close to one.

Fast solver for large-scale time-harmonic Maxwell's equations in SiP applications

Shaoliang Hu(胡少亮)
Institute of Applied Physics and Computational Mathematics

System in package (SiP) is mainstream technology in the design of electronics system. Numerical simulation plays an important role in SiP applications. However, due to the specific complexity of SiP applications, existing algorithms for linear systems arising from time-harmonic Maxwell's equations are faced with great challenges, which become a bottleneck restricting efficiency of large-scale numerical simulations. In this talk, we will introduce the application features of SiP, and evaluate the capability of existing algorithms for realistic SiP models. Based on these, we propose a preconditioning strategy and demonstrate its feasibility and efficiency in solving large-scale linear systems originated from filter and RFFE models. Furthermore, we analyze impact of such applications on performance behavior of current algorithms and the challenges we faced with.

The acoustic half space Green's function with impedance boundary condition in d spatial dimensions: Fast evaluation and numerical quadrature

Stefan A. Sauter
Universität Zürich

In our talk, we introduce a new representation of the acoustic half space Green's function with impedance boundary conditions in d space dimensions which avoids oscillatory Fourier integrals and asymptotic expansions. A numerical quadrature method is developed for its fast evaluation.
In the context of boundary element methods this function must be integrated over pairs of simplices and we present an efficient approximation method.

Hybrid numerical-asymptotic boundary element methods for high frequency scattering by multiple screens

Stephen Langdon
Brunel University, London

Standard Boundary Element Methods (BEM) for time-harmonic wave scattering problems, with piecewise polynomial approximation spaces, can be prohibitively expensive when the wavelength of the scattered wave is small compared to typical length scales of the scatterer, due to rapid oscillations in the solution. Hybrid Numerical-Asymptotic (HNA) BEMs, with enriched approximation spaces consisting of the products of piecewise polynomials with carefully chosen oscillatory basis functions, informed by high frequency asymptotics, have been shown to be effective in overcoming this limitation for a range of problems, mostly focused on single convex scatterers or very specific non-convex or multiple scattering configurations. Here we consider extension of the HNABEM approach to more general multiple scattering problems, focusing in particular on two-dimensional scattering by multiple screens.

Preconditioning the EFIE on Screens

Ralf Hiptmair
ETH Zurich

We consider the electric field integral equation (EFIE) modeling the scattering of time-harmonic electromagnetic waves at a perfectly conducting screen. When discretizing the EFIE by means of low-order Galerkin boundary methods (BEM), one obtains linear systems that are ill-conditioned on fine meshes and for low wave numbers k. This makes iterative solvers perform poorly and entails the use of preconditioning.

In order to construct optimal preconditioners for the EFIE on screens, the authors recently derived compact equivalent inverses of the EFIE operator on simple Lipschitz screens in [R. Hiptmair and C. Urzúa-Torres, Compact equivalent inverse of the electric field integral operator on screens, Integral Equations Operator Theory 92 (2020) 9]. This paper elaborates how to use this result to build an optimal operator preconditioner for the EFIE on screens that can be discretized in a stable fashion. Furthermore, the stability of the preconditioner relies only on the stability of the discrete L2 duality pairing for scalar functions, instead of the vectorial one. Therefore, this novel approach not only offers h-independent and k-robust condition numbers, but it is also easier to implement and accommodates non-uniform meshes without additional computational effort.

This is joint work with Carolina Urzúa-Torres, TU Delft

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